INTERNATIONAL CONFERENCE ON THE DYNAMICS OF COASTAL ZONE OF NON-TIDAL SEAS

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WAVE DEFORMATION AND BREAKING IN THE COASTAL ZONE

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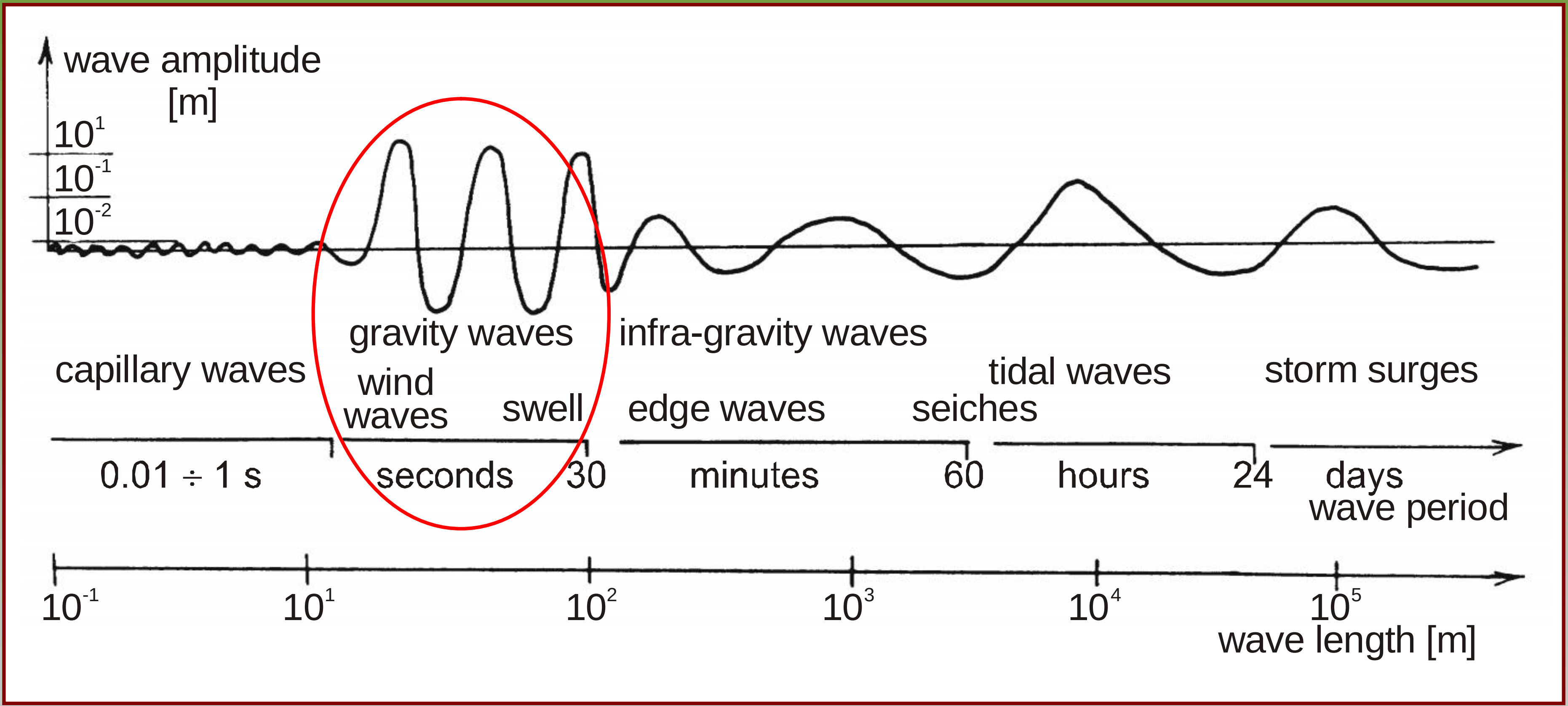
- INTRODUCTION
- BASIC WAVE TRANSFORMATION PROCESSES
- WAVE THEORIES / SHAPES
- WAVE ENERGY DISSIPATION / BREAKING
- IMPORTANCE OF THE ISSUE – ILLUSTRATIVE EXAMPLES

  a) wave energy dissipation; b) wave shape

Applications of wave modelling/forecasting:

- shore protection
- maritime construction (e.g. harbour) design
- navigation (hydro-meteorological services)
- marine ecology
- exploitation of underwater resources

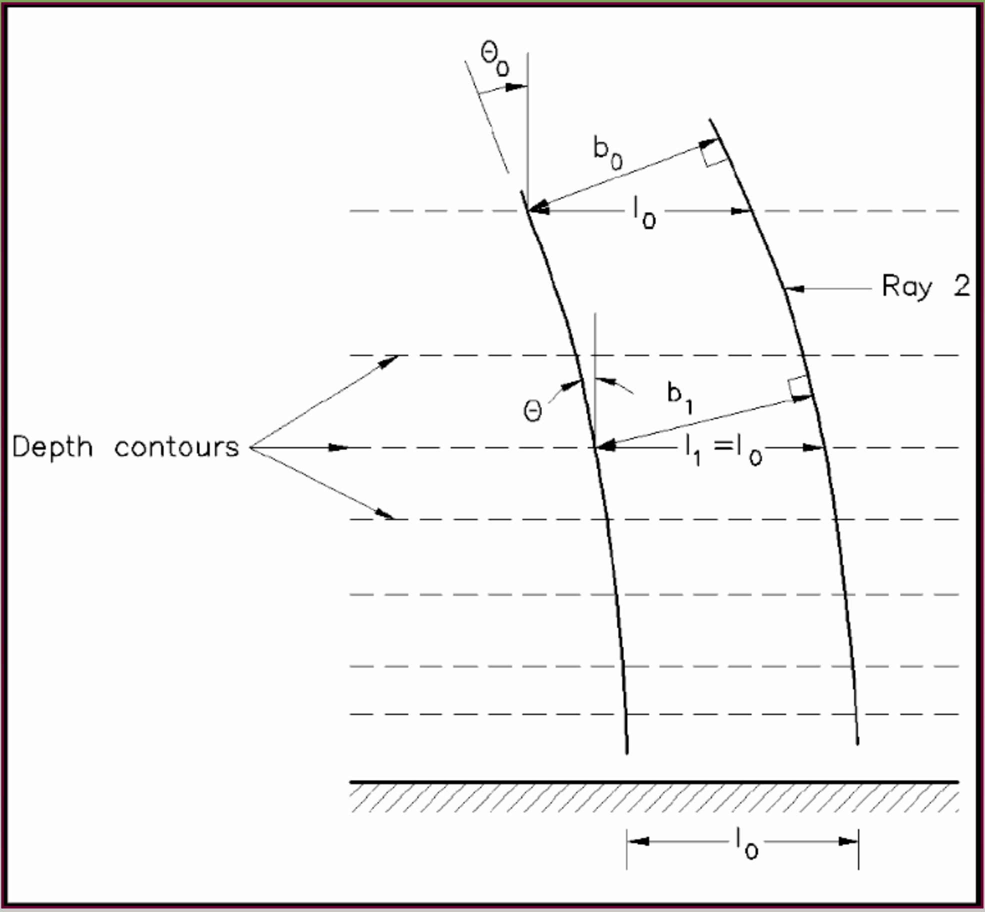


Time and spatial domains of various surface waves in the coastal zone

Processes affecting a wave during propagation from deep into shallow water:

- refraction
- shoaling
- diffraction
- reflection
- dissipation due to friction
- dissipation due to percolation
- dissipation due to breaking
- additional growth due to the wind
- wave-current interaction
- wave-wave interactions

Wave refraction and shoaling



Snell law: $\frac{\sin\theta}{C} = \frac{\sin\theta_0}{C_0}$

Conservation of energy flux:

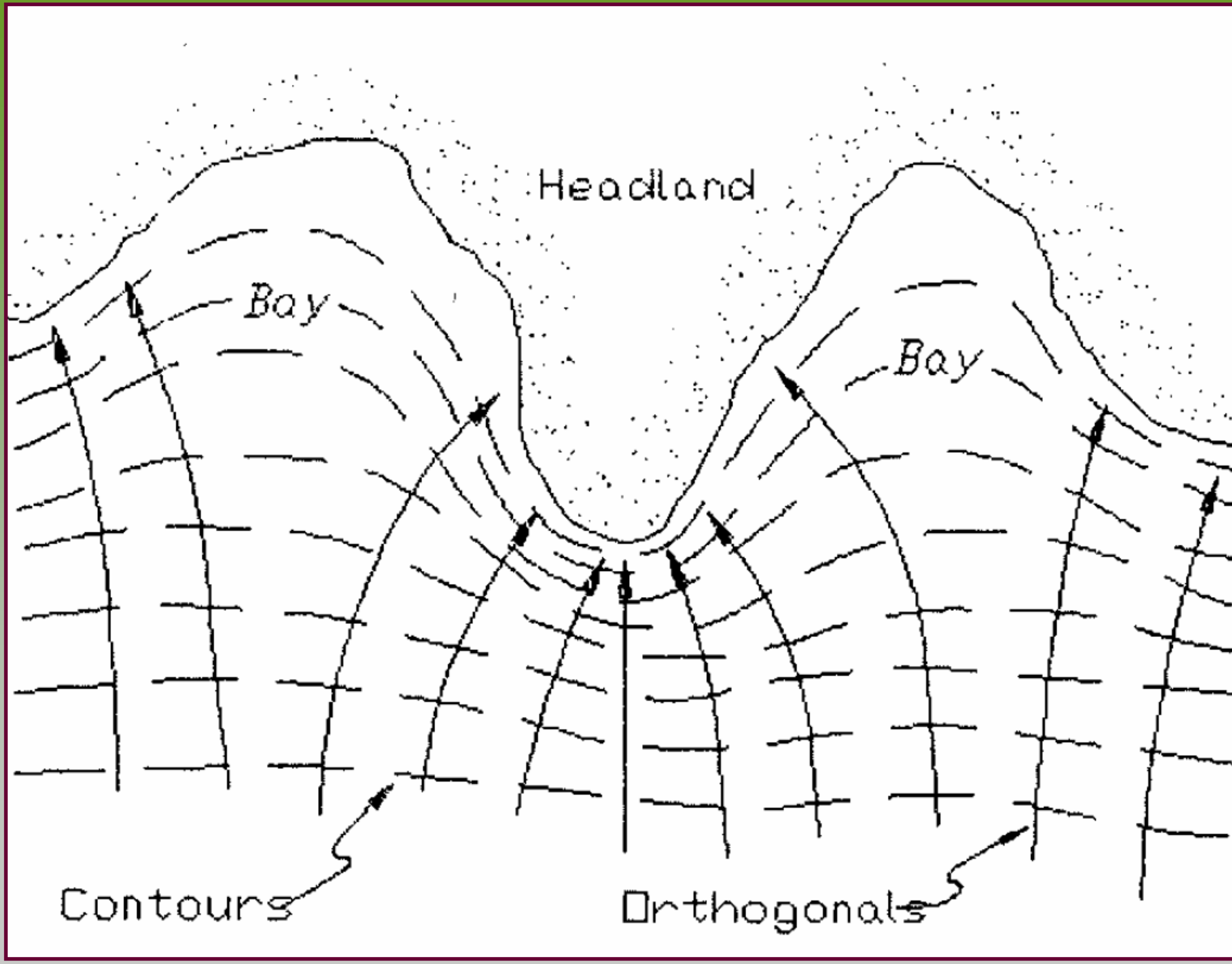
$$(EC_{g0})b_0 = (EC_{g1})b_1$$

$$E = \frac{1}{8}\rho g H^2$$

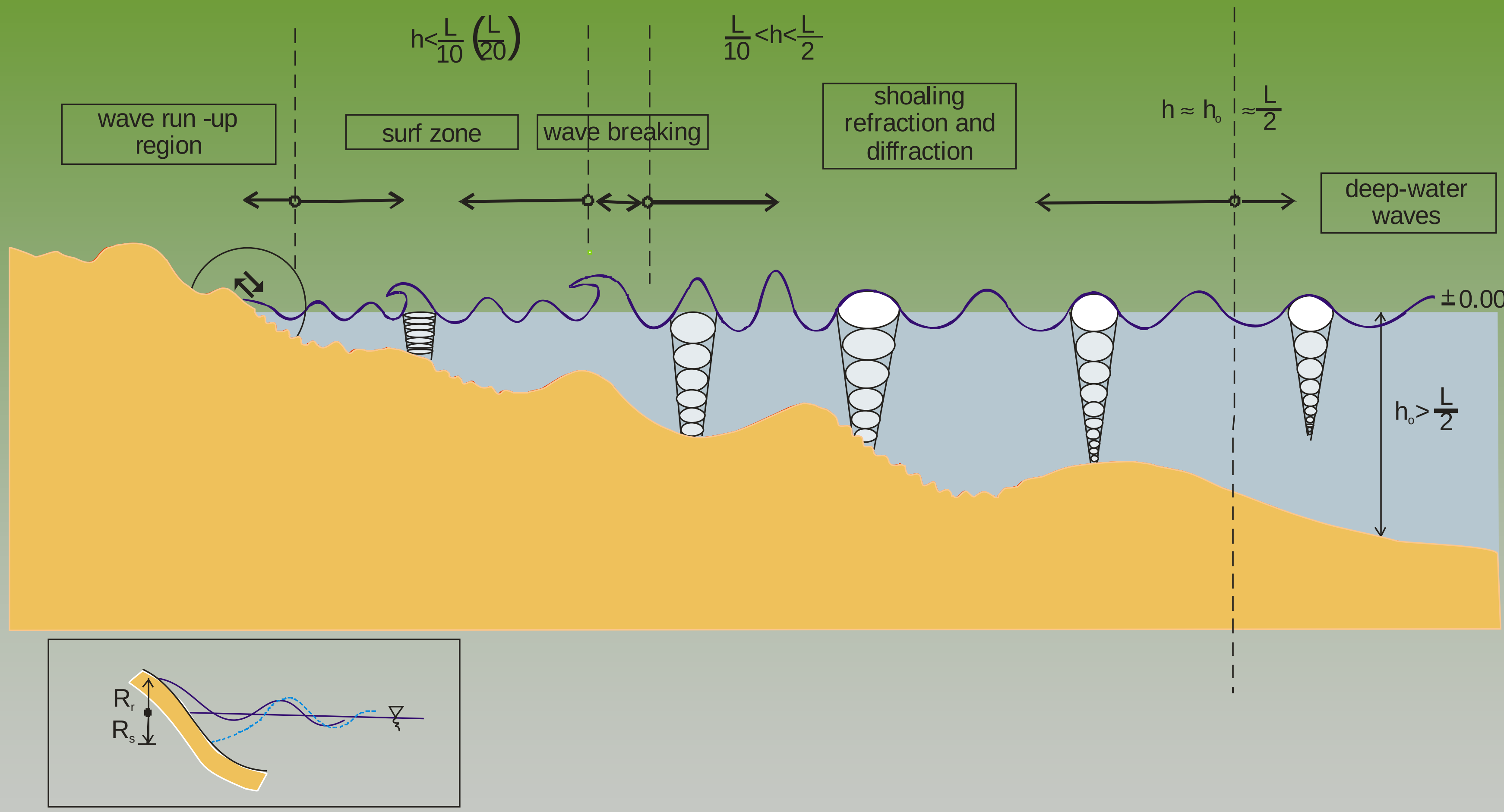
$$H_1 = H_0\sqrt{\frac{C_{g_0}}{C_{g_1}}}\sqrt{\frac{b_0}{b_1}}$$

$$H_1 = H_0 K_s K_r$$

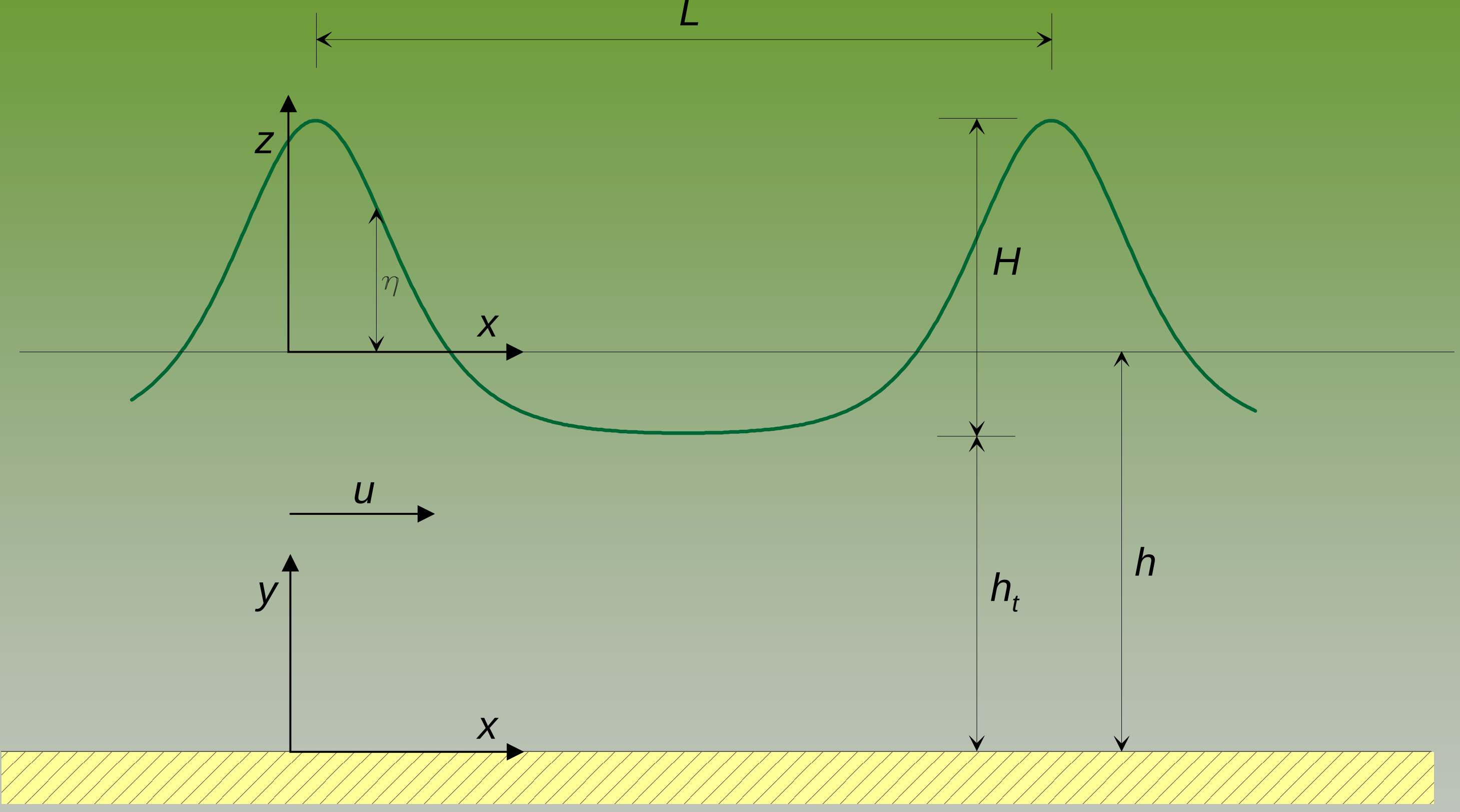
$$K_r = \left(\frac{b_0}{b_1}\right)^{\frac{1}{2}} = \left(\frac{\cos\theta_0}{\cos\theta_1}\right)^{\frac{1}{2}} = \left(\frac{1-\sin^2\theta_0}{1-\sin^2\theta_1}\right)^{\frac{1}{4}}$$



Exemplary scheme of wave refraction



Wave transformation in the coastal zone



Definition sketch for wave theories

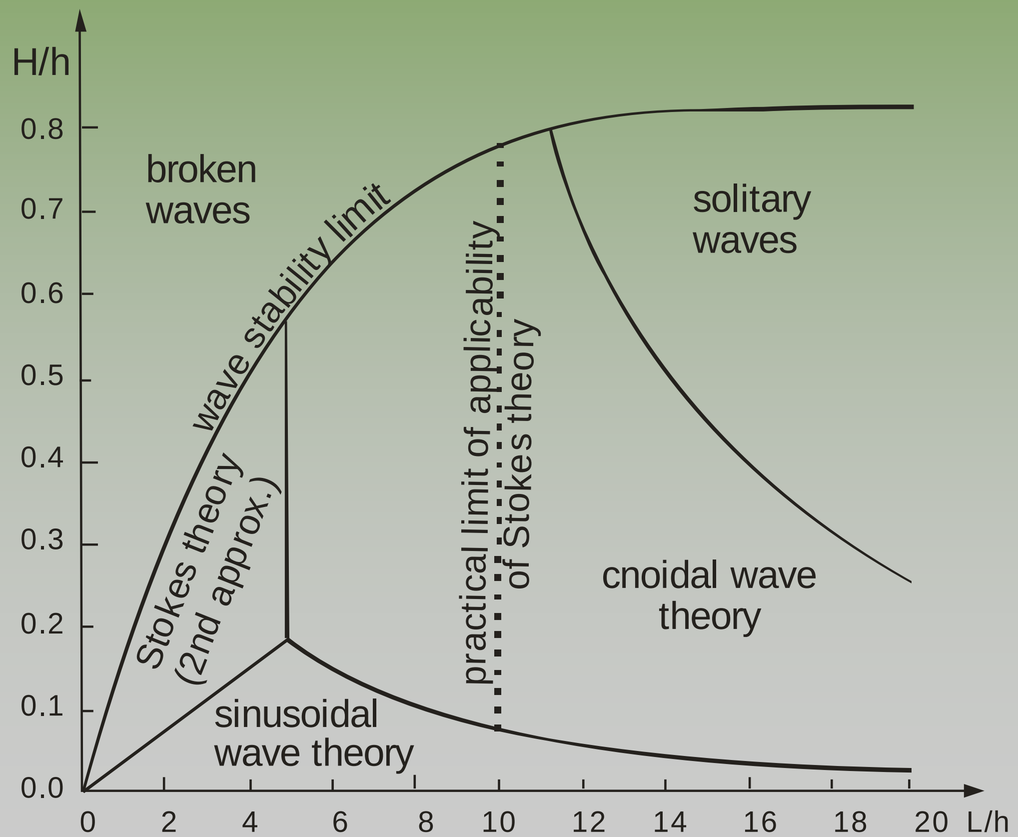
Ursell parameter: $U_r = H/h(L/h)^2$

Wave length to water depth ratio: L/h

Waves of small amplitude ➔ sinusoidal (linear) wave theory

$U_r < 20$ & $L/h < 8$ ($L/h < 10$) ➔ Stokes approximations

$U_r > 20$ & $L/h > 8$ ($L/h > 10$) ➔ cnoidal wave theory



Applicability ranges of wave theories

Sinusoidal (linear) wave theory

Free surface elevation: $$\eta(x,t) = \frac{H}{2}\cos\left[2\pi\left(\frac{x}{L} - \frac{t}{T}\right)\right]$$

Horizontal velocity: $$u(x,z,t) = \frac{gHT}{2L}\,\frac{\cosh\left(\frac{2\pi(z+h)}{L}\right)}{\cosh\left(\frac{2\pi h}{L}\right)}\cos\left[2\pi\left(\frac{x}{L} - \frac{t}{T}\right)\right]$$

H – wave height
T – wave period
h – water depth
L – wave length

Dispersion relationship: $$\left(\frac{2\pi}{T}\right)^2 = \frac{2\pi}{L}\,g\tanh\left(\frac{2\pi}{L}h\right)$$

Stokes wave theory (2nd order approximation)

Free surface elevation:

$$\eta(x,t) = \frac{H}{2}\cos\left[2\pi\left(\frac{x}{L}-\frac{t}{T}\right)\right]+$$

$$\left(\frac{\pi H^2}{8L}\right)\frac{\cosh\left(\frac{2\pi h}{L}\right)}{\sinh^3\left(\frac{2\pi h}{L}\right)}\left[2+\cosh\left(\frac{4h}{L}\right)\right]\cos\left[4\pi\left(\frac{x}{L}-\frac{t}{T}\right)\right]$$

Horizontal velocity:

$$u(x,z,t) = \frac{gHT}{2L}\frac{\cosh\left(\frac{2\pi(z+h)}{L}\right)}{\cosh\left(\frac{2\pi h}{L}\right)}\cos\left[2\pi\left(\frac{x}{L}-\frac{t}{T}\right)\right]+$$

$$\frac{3}{4}\left(\frac{\pi H}{L}\right)^2 C\,\frac{\cosh\left(\frac{4\pi(z+h)}{L}\right)}{\sinh^4\left(\frac{2\pi h}{L}\right)}\cos\left[4\pi\left(\frac{x}{L}-\frac{t}{T}\right)\right]$$

Cnoidal wave theory

Free surface elevation: $$\eta(x,t) = h_t + H\,\mathrm{cn}^2(x,t,k)$$

$$h_t = h\left\{1 + \frac{H}{k^2 h}\left[1 - k^2 - \frac{\mathbf{E}(k)}{\mathbf{K}(k)}\right]\right\}$$

$$\mathrm{cn}^2(x,t,k) = \mathrm{cn}^2\left[2\mathbf{K}(k)\left(\frac{x}{L} - \frac{t}{T}\right); k\right]$$

‘cn’ – Jacobian elliptic cosine (for *k*=1: solitary wave, for *k*=0: sinusoidal wave)
k – modulus of elliptic integrals

K(*k*), **E**(*k*) – complete elliptic integrals of the first and second kind, respectively

$$\left(\frac{H}{h}\right)\left(\frac{gT^2}{h}\right) = \frac{16}{3}k^2\mathbf{K}^2(k)$$

Cnoidal wave length (assuming $C=L/T=(gh)^{1/2}$): $$\left(\frac{H}{h}\right)\left(\frac{L}{h}\right)^2 = \frac{16}{3}k^2\mathbf{K}^2(k)$$

Cnoidal wave theory (continued)

Sobey et al. (1987)

Horizontal velocity: $$u(x,t)=\bar{u}+(gh_t)^{1/2}\left[-1-\frac{H}{h_t\,\mathrm{cn}^2(x,t,k)}\left(-0.5+k^2-k^2\mathrm{cn}^2(x,t,k)\right)\right]$$

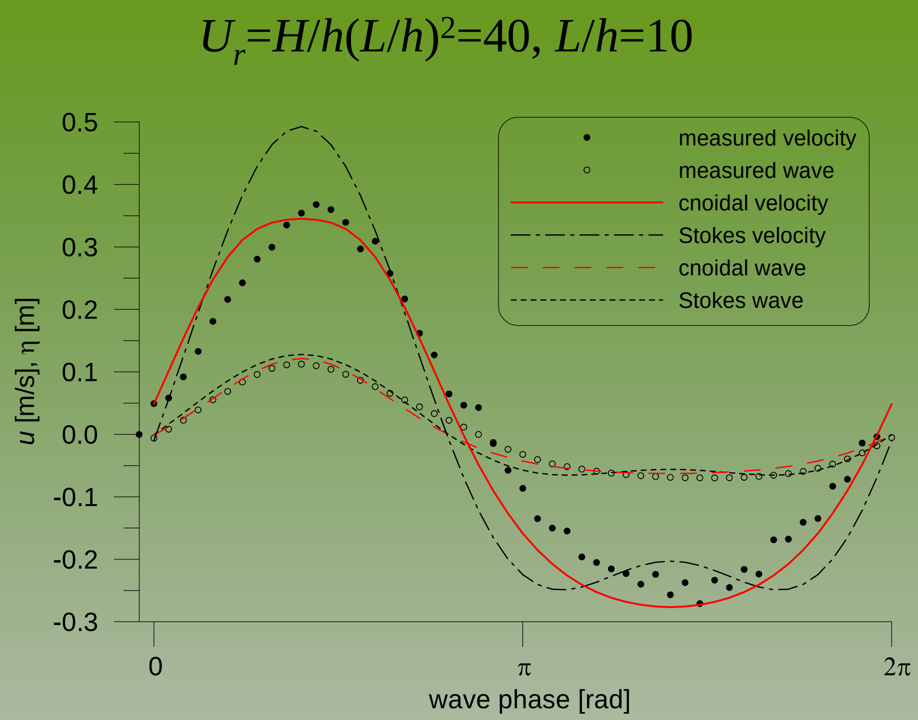
$$\bar{u}=(gh_t)^{1/2}\left\{1+\frac{H}{k^2h_t}\left[0.5-\frac{\mathbf{E}(k)}{\mathbf{K}(k)}\right]\right\}$$

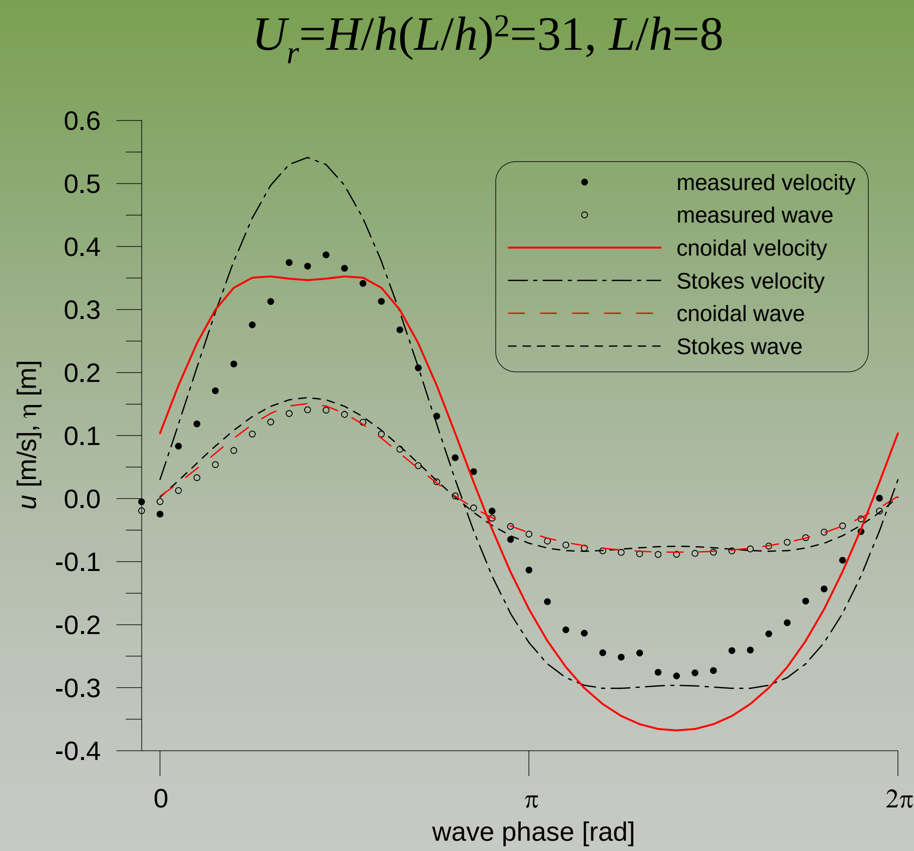
Wiegel (1960)

$$\frac{u(x,y,t)}{(gh)^{1/2}}=-\frac{5}{4}+\frac{3h_t}{2h}-\frac{h_t^2}{4h^2}+\left(\frac{3h}{2h}-\frac{h_tH}{2h^2}\right)\mathrm{cn}^2(x,t,k)-\frac{H^2}{4h^2}\mathrm{cn}^4(x,t,k)$$

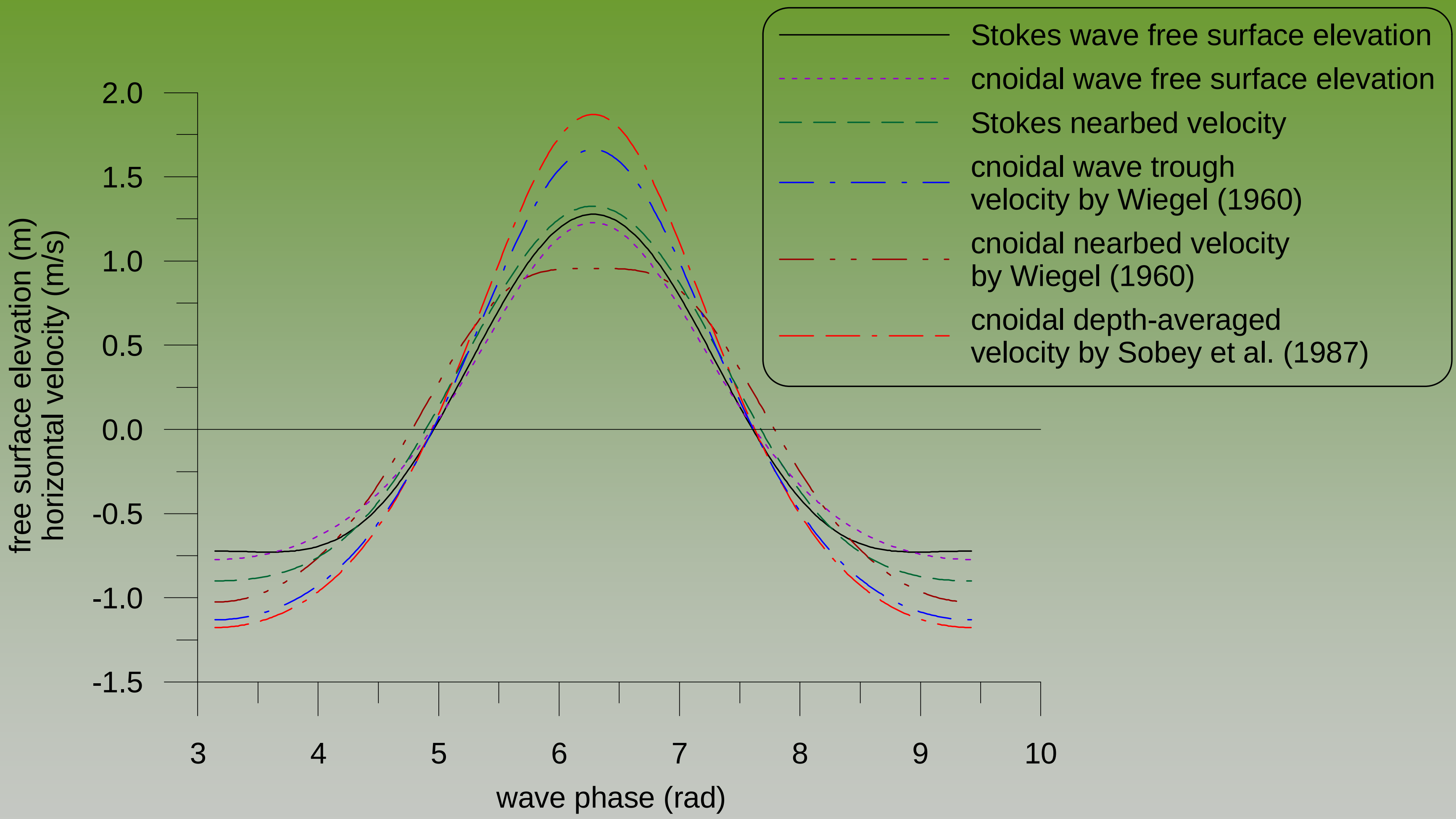
Horizontal velocity: $$-\frac{8H\mathbf{K}^2(k)}{L^2}\left(\frac{h}{3}-\frac{y^2}{2h}\right)\Big[-k^2\mathrm{sn}^2(x,t,k)\mathrm{cn}^2(x,t,k)+\mathrm{cn}^2(x,t,k)\mathrm{dn}^2(x,t,k)$$
$$-\mathrm{sn}^2(x,t,k)\mathrm{dn}^2(x,t,k)\Big]$$

‘sn’, ‘dn’ – Jacobian elliptic functions ($\mathrm{sn}^2+\mathrm{cn}^2=1$, $k^2\mathrm{sn}^2+\mathrm{dn}^2=1$)





Free surface elevation and nearbed velocities by cnoidal theory and 2nd Stokes approximation versus IBW PAN laboratory data



Free surface elevation and wave-induced velocity by various approaches for h=5 m, H=2 m, T=6 s; L/h≈8, U_r≈23

Wave breaking criteria and breaker types

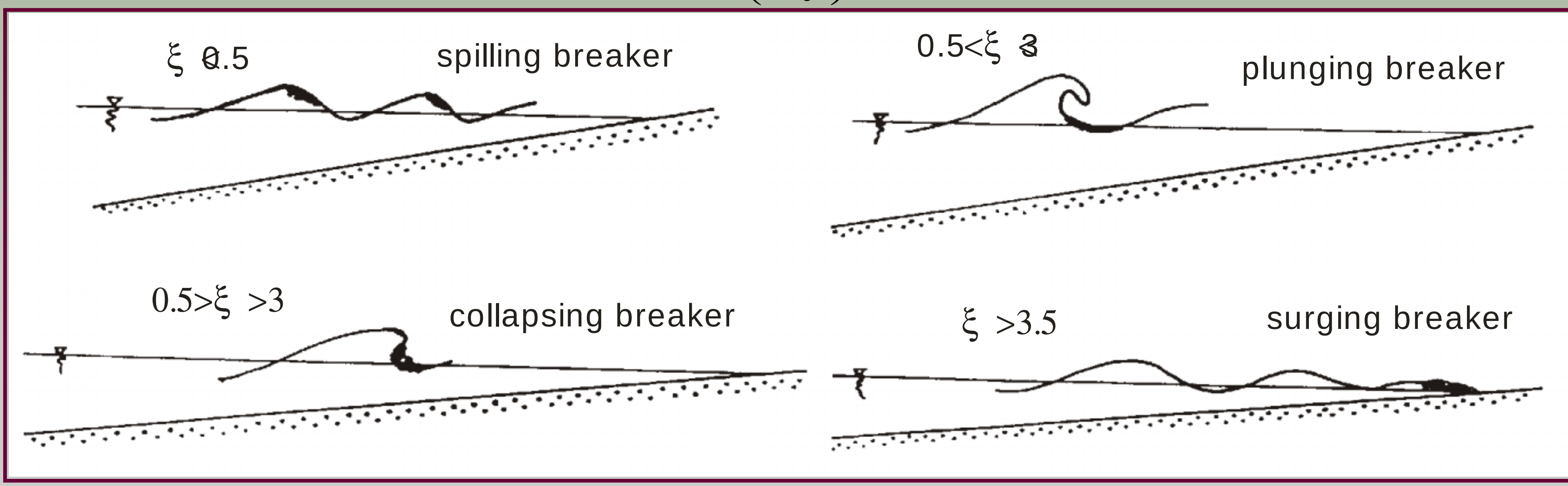
Kinematic criterion: $u_{crest} > C$ (local horizontal orbital velocity exceeds phase velocity)

Breaker depth index: $\gamma_b = H_b / h_b$

$\gamma_b = 0.4$ (”saturated” breaking zone for irregular waves), 0.78 (solitary wave)

$$\gamma_b = 0.5 + 0.4 \tanh\left(33 \frac{H_{rms,0}}{L_{p,0}}\right) \qquad \gamma_b = 0.8 \quad \text{(Baltic Sea)}$$

Surf similarity parameter: $\xi_0 = \tan\beta \left(\frac{H_0}{L_0}\right)^{-\frac{1}{2}}$



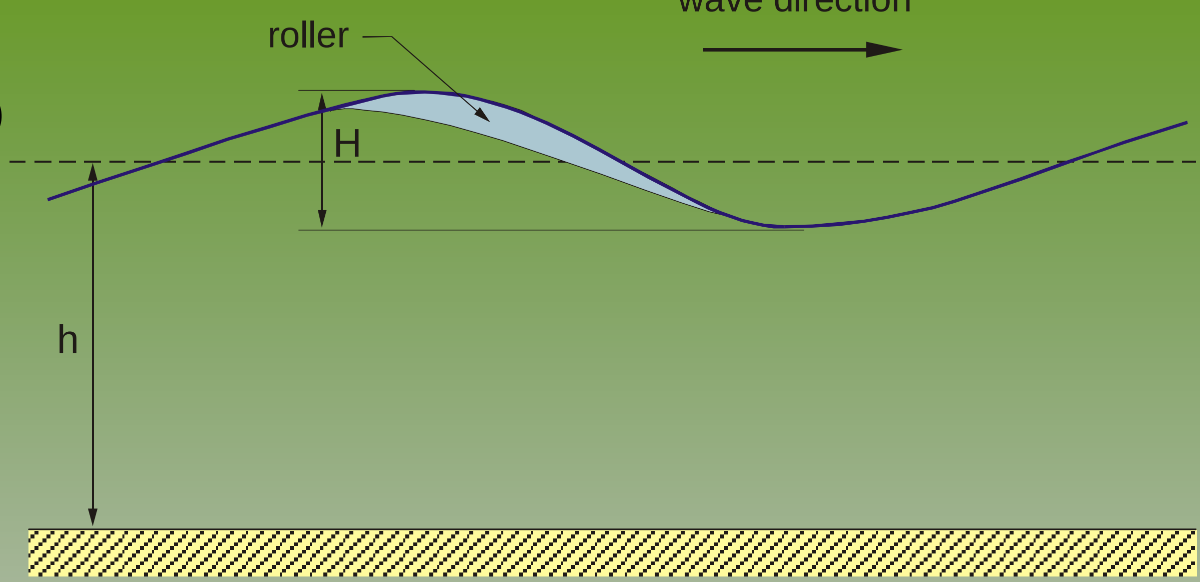
Spilling breaker

Plunging breaker

White-capping

BREAKING WAVE

Battjes & Janssen (1978)
Svendsen (1984)
Szmytkiewicz & Skaja (1991)
Szmytkiewicz (2002)
Janssen & Battjes (2007)



- linear wave refraction
- Snell law
- dispersion relationship for the linear wave theory
- no wave reflections from the shore
- narrow spectrum of random waves in the coastal zone
- Rayleigh distribution of the wave height
- equation of wave energy flux conservation
- wave energy dissipation due to breaking
- criterion of wave breaking by Miche
- isobaths approximately parallel to the shoreline
- fully developed roller in front of breaking wave crest

Computation of wave height H

Equation of wave energy flux conservation:

$$\frac{\partial}{\partial x}(EC_g\cos\theta)+\frac{\partial}{\partial x}(E_r C\cos\theta)=-D$$

E – total wave energy
E_r – kinetic energy of the roller (as described by Svendsen 1984)
C – phase velocity of waves
C_g – group velocity of waves
θ – wave approach angle
D – wave energy dissipation

$$D=\frac{\alpha}{4}p_b f_p \rho g H_{\max}^2$$ (Battjes & Janssen 1978)

g – acceleration due to gravity
ρ – water density
α – empirical coefficient of the order O(1)
f_p – wave spectrum peak frequency ($f_p=1/T_p$)
p_b – factor characterising the percentage of broken and breaking waves at a given point in the coastal zone, described as:

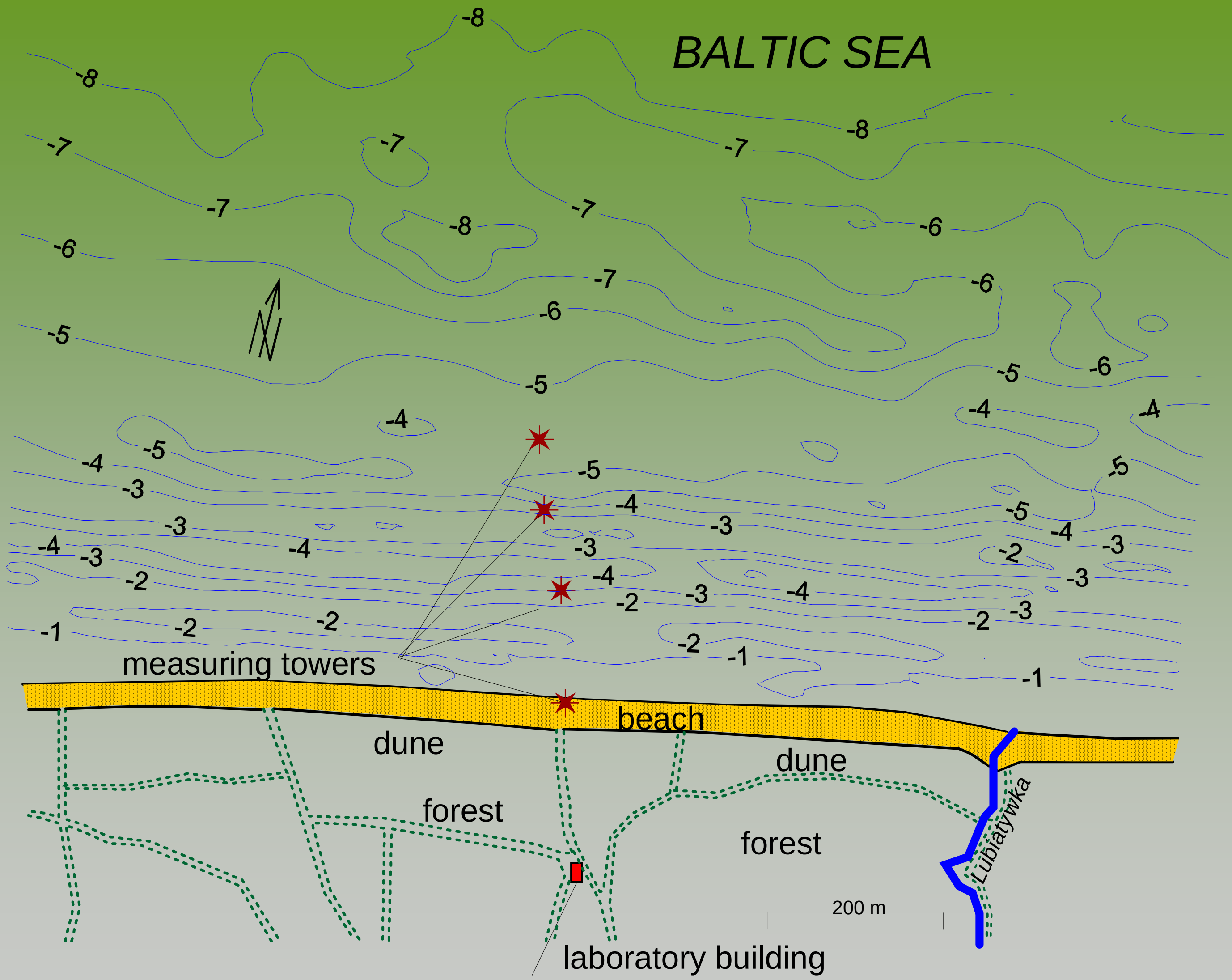
$$\frac{1-p_b}{\ln p_b}=-\left(\frac{H_{rms}}{H_{\max}}\right)^2$$

$H_{\max}$ – maximum possible wave height at the considered location of the coastal zone
H_{rms} – sought root-mean-square wave height

$$H_{\max}=0.88k_p^{-1}\tanh(\gamma_b k_p h/0.88)$$ (criterion by Miche)

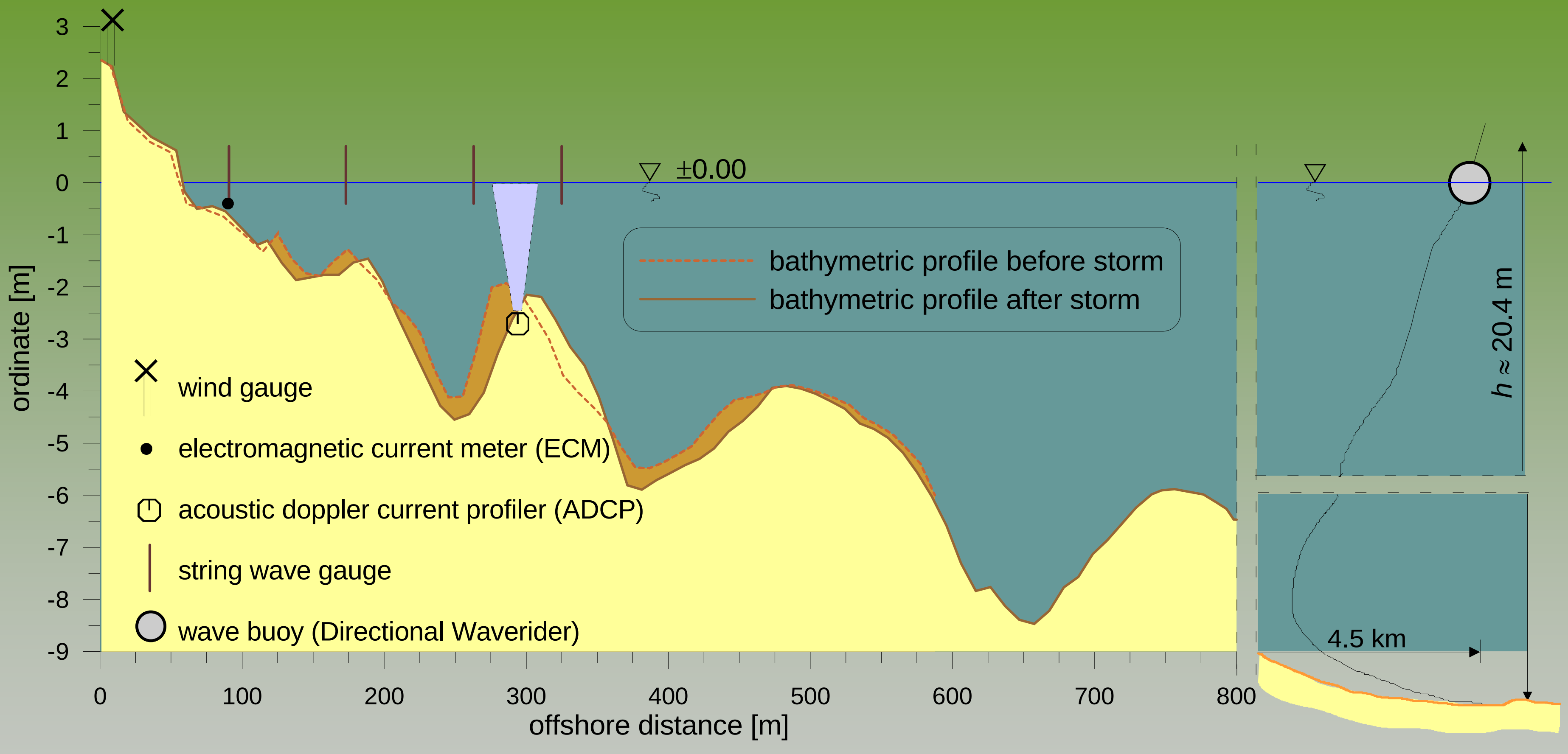
k_p – wave number calculated from the dispersion relationship with the wave spectral peak f_p
γ_b – empirical coefficient of wave breaking

Location of the IBW PAN Coastal Research Station (CRS) in Lubiatowo

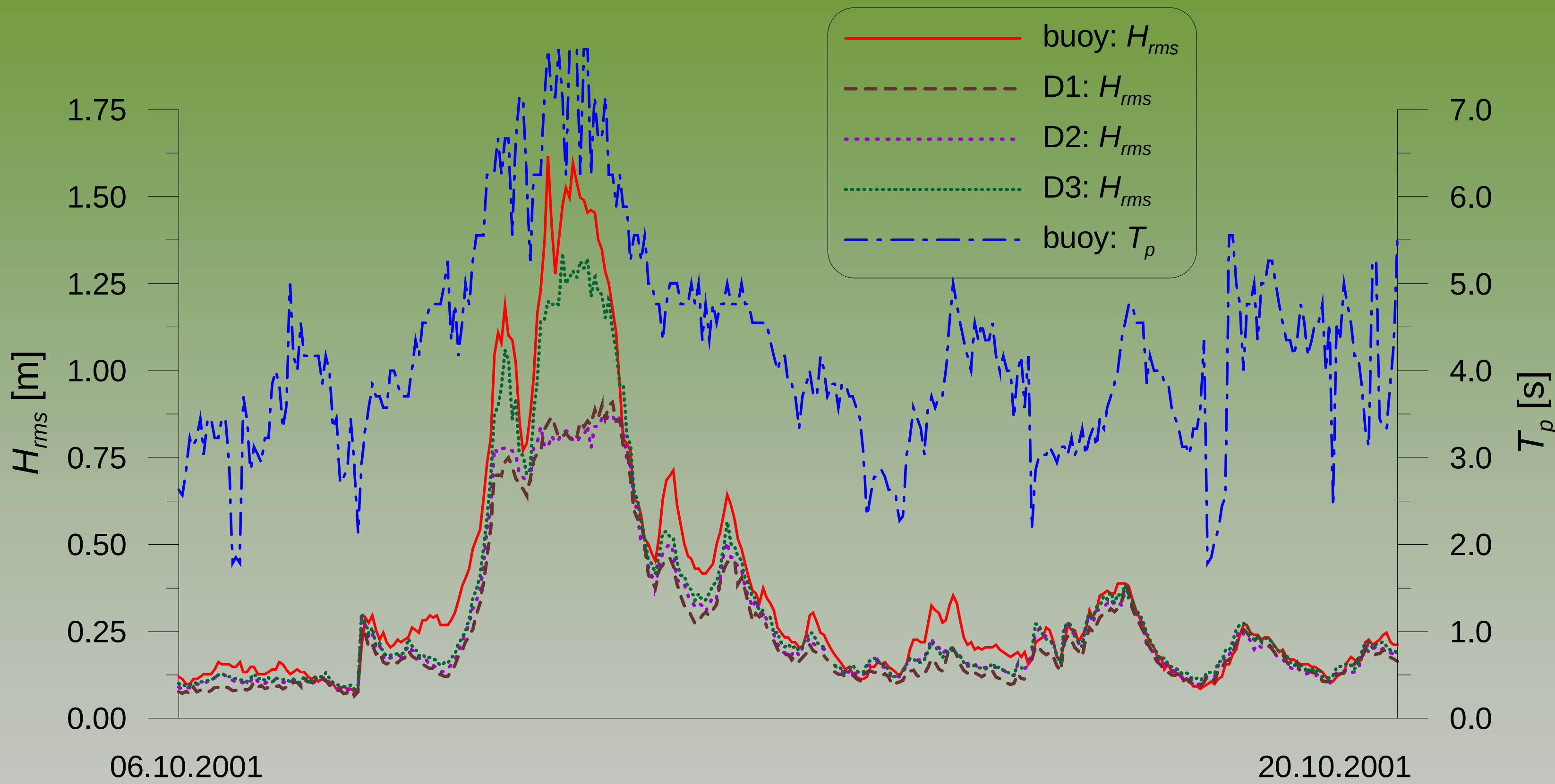


Layout of Lubiatowo site

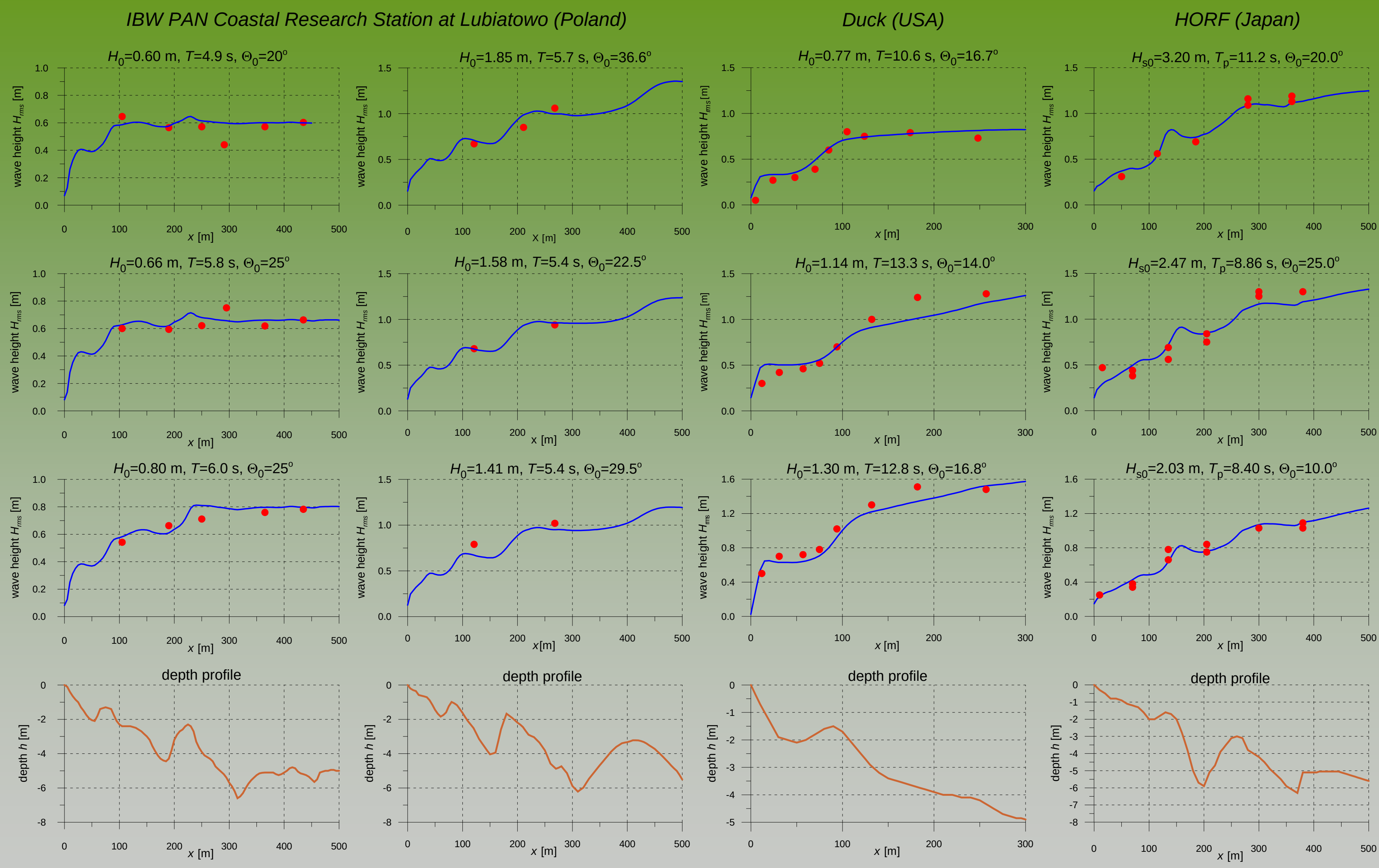
Laboratory building and measuring towers of CRS Lubiatowo



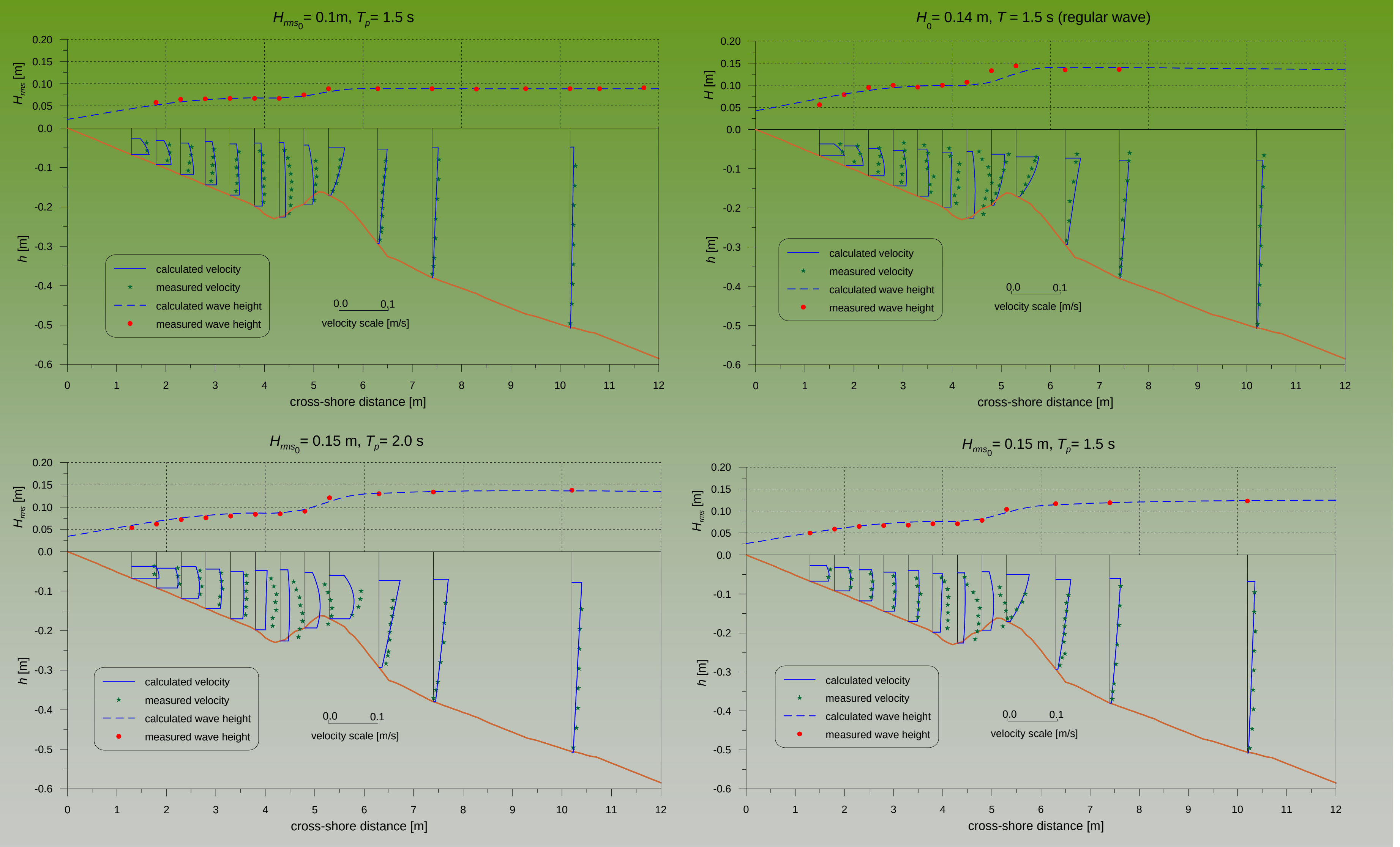
Exemplary layout of measuring equipment on the cross-shore transect at CRS Lubiatowo



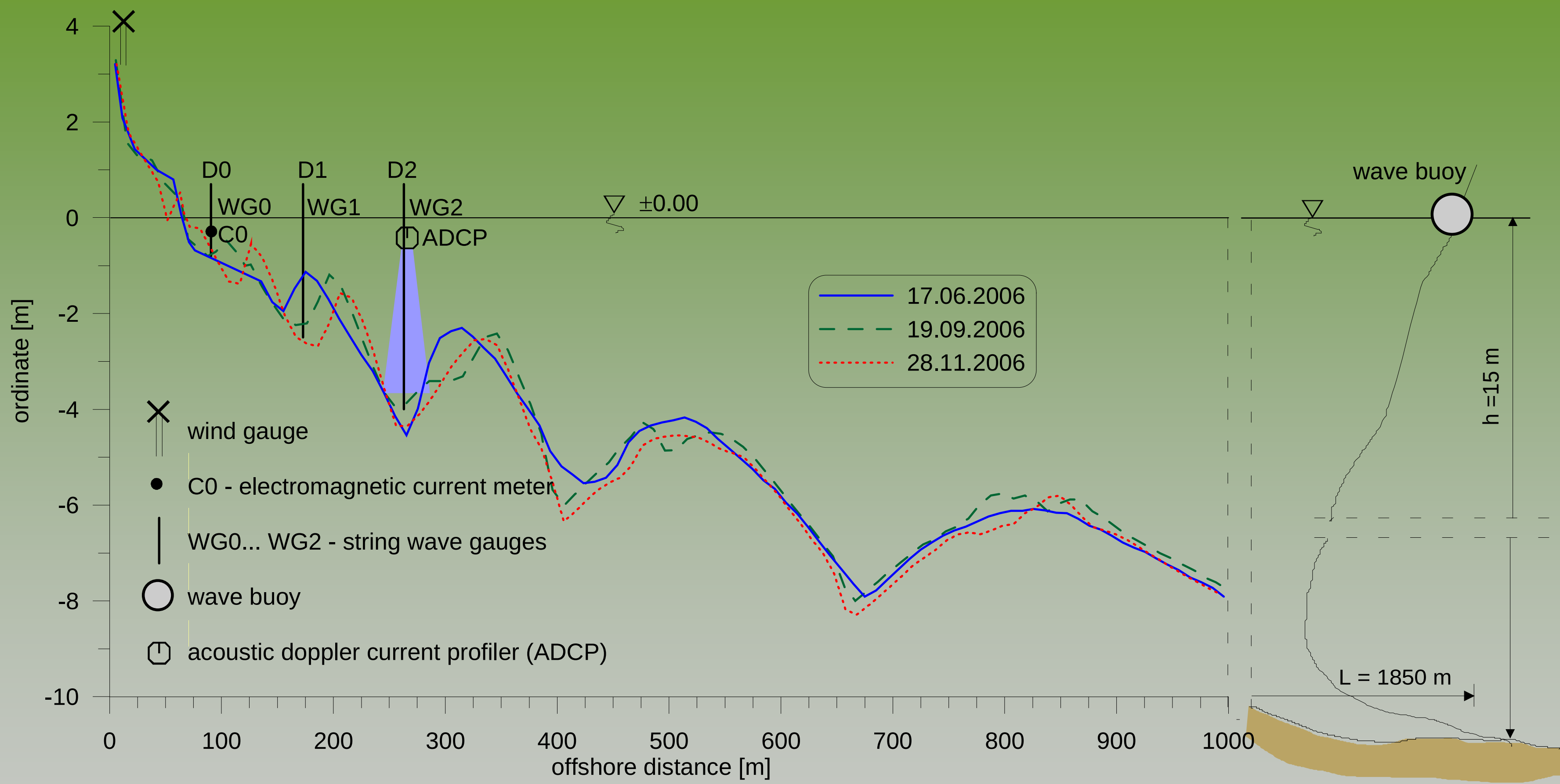
Exemplary wave records at deep water and at measuring towers



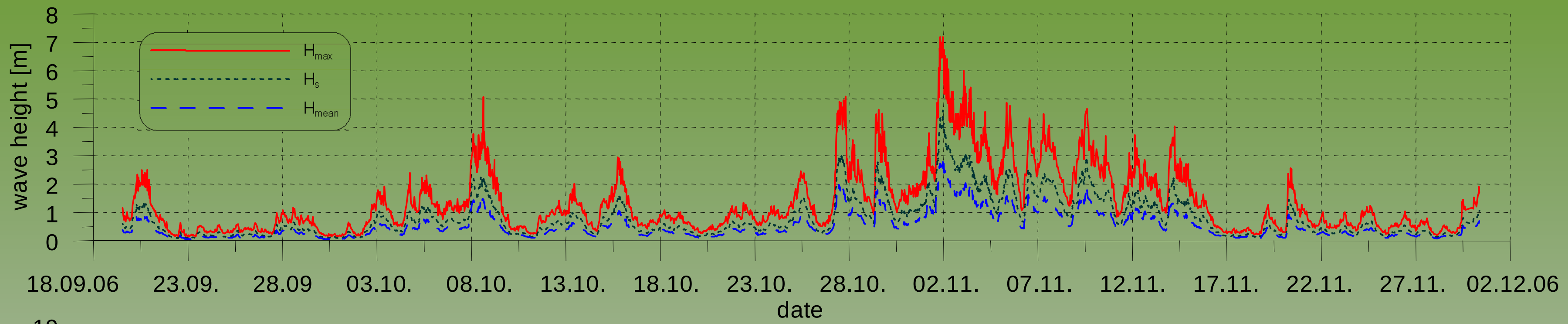
Variability of wave height on the cross-shore profile - comparisons after Szmytkiewicz (2002)

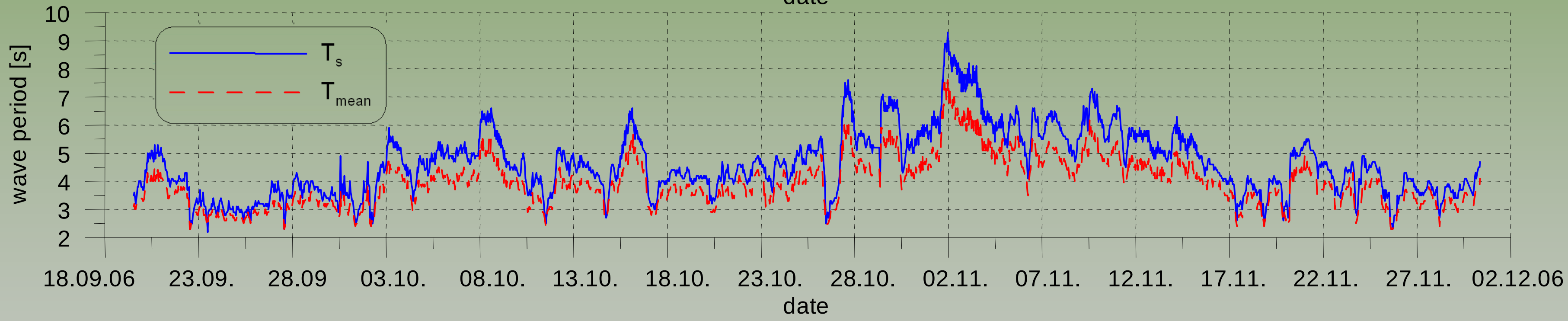


Wave height and undertow velocity (IBW PAN wave flume data) - comparisons after Szmytkiewicz (2002)

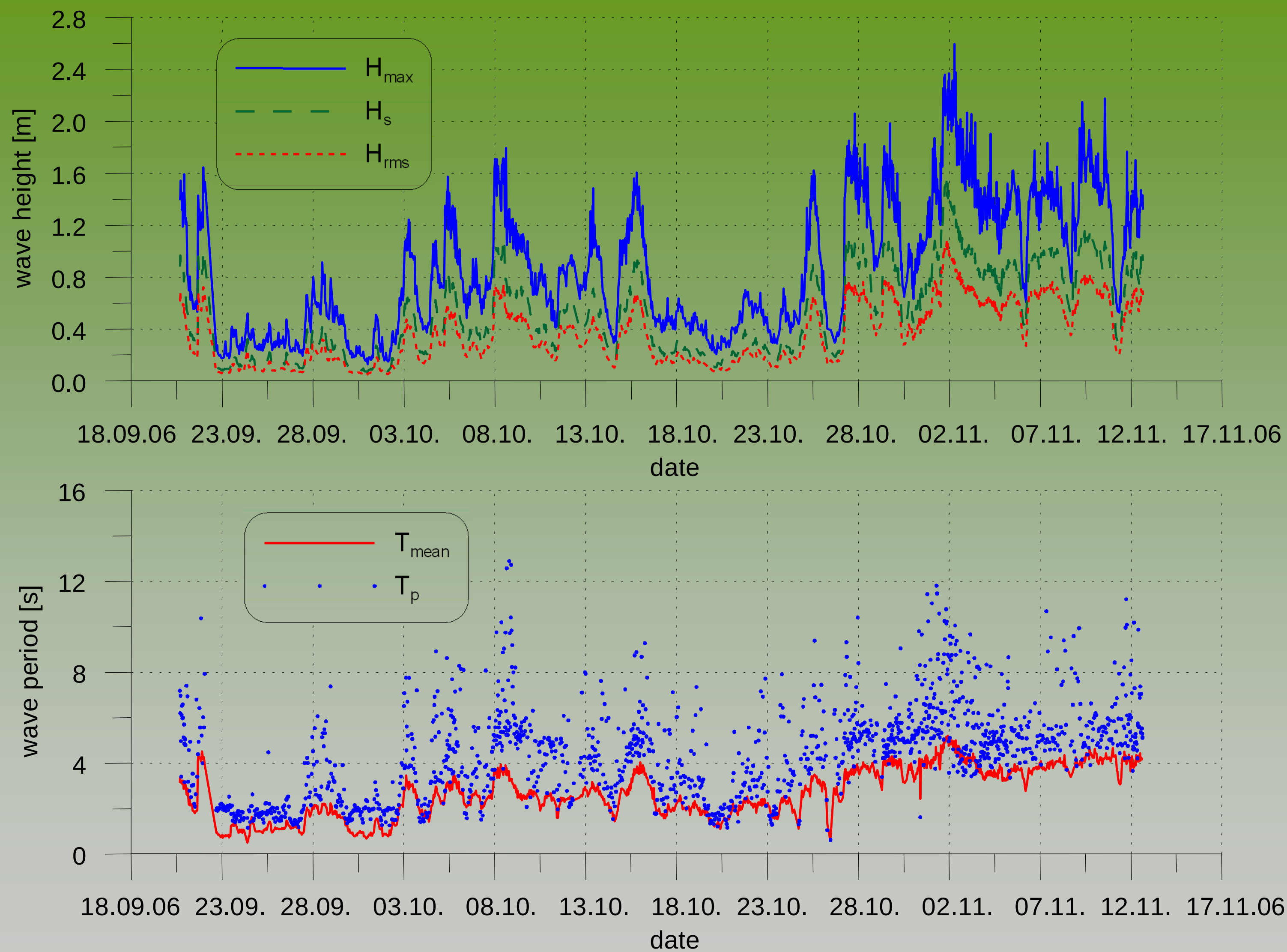


Measuring devices on the cross-shore profile during field experiment Lubiatowo 2006

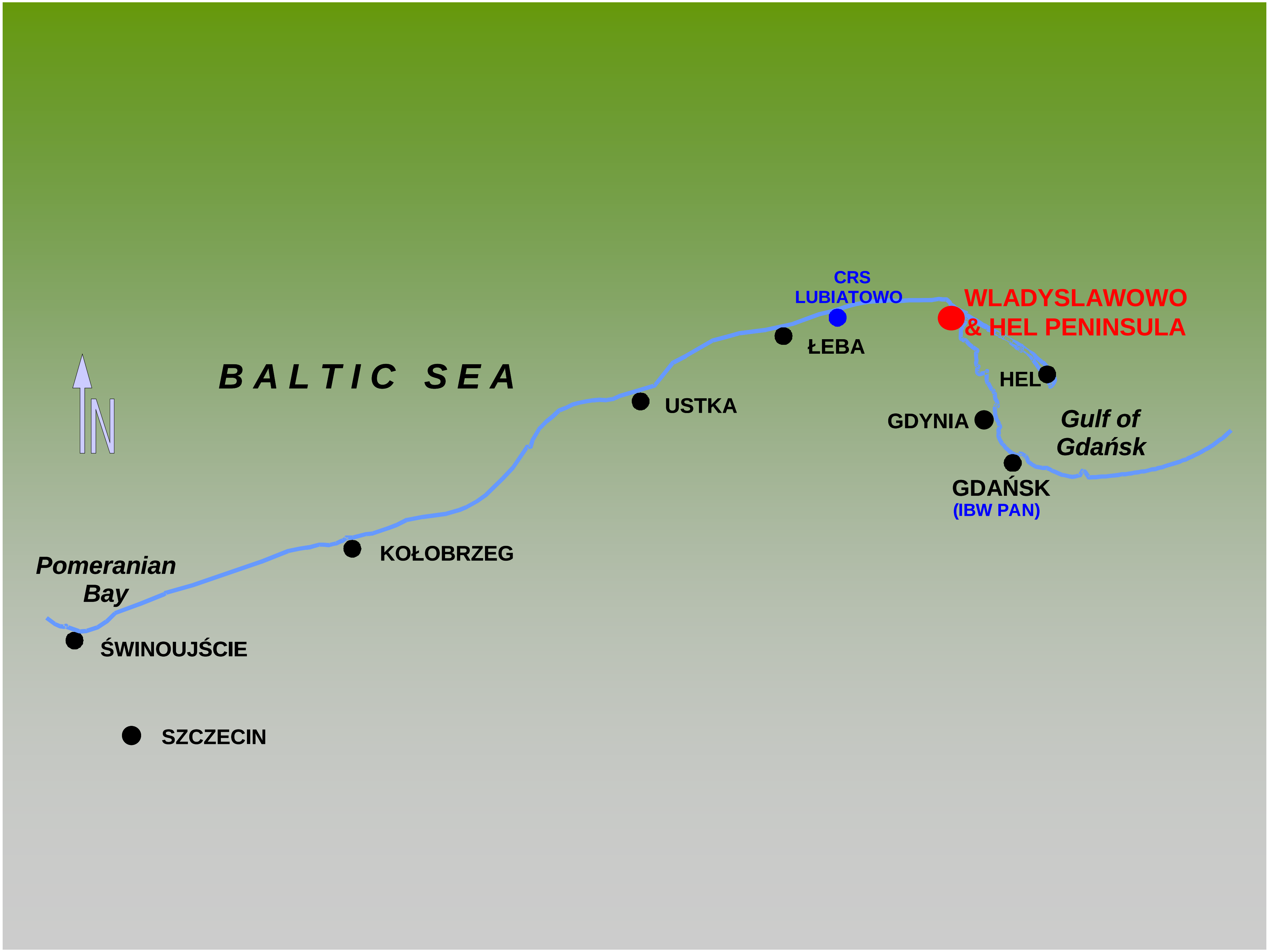




Offshore wave heights and periods registered at CRS Lubiatowo from 19 Sep. to 30 Nov. 2006

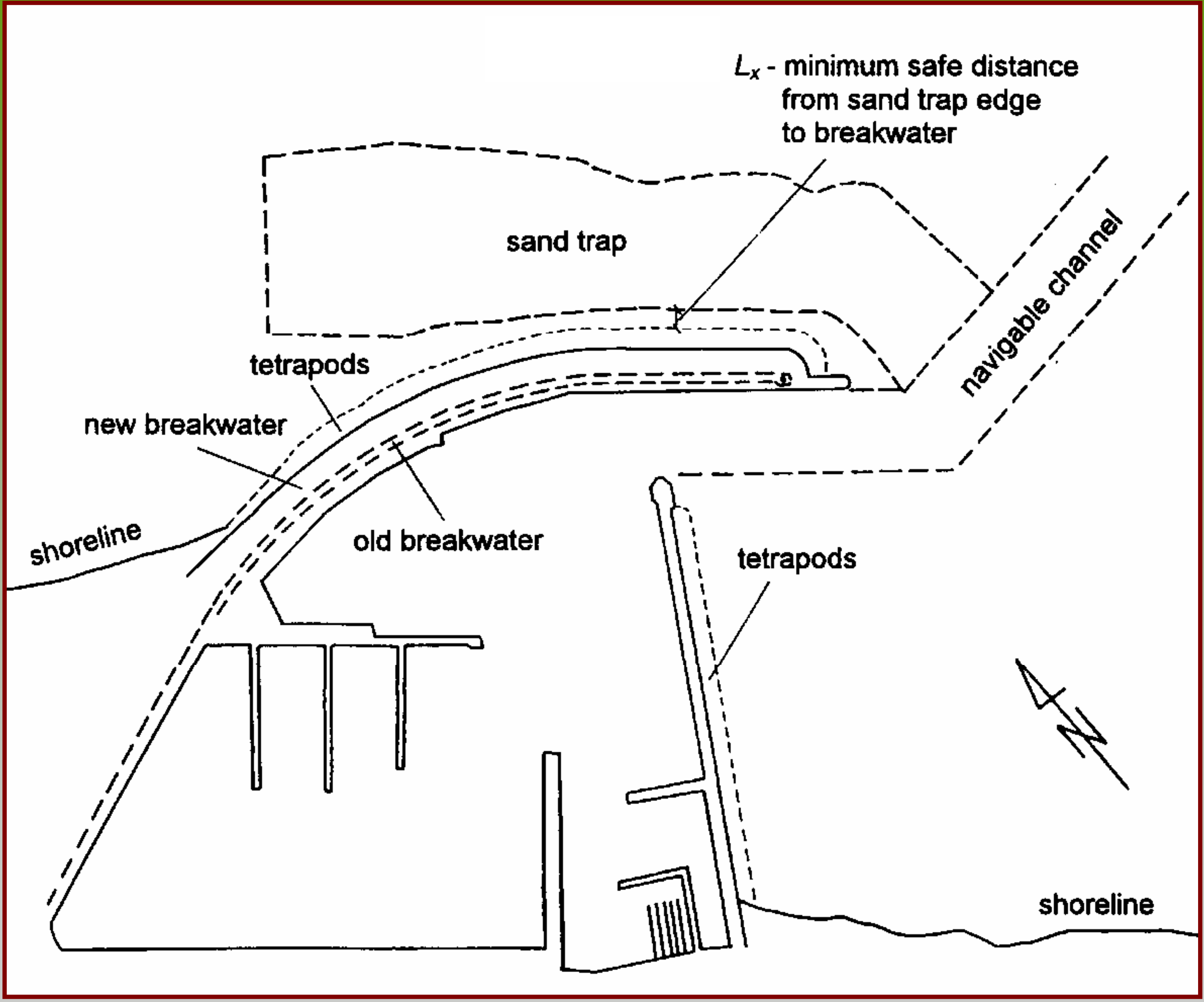


Shallow-water (h = 4.4 m, location D2) wave heights and periods registered at CRS Lubiatowo in autumn 2006



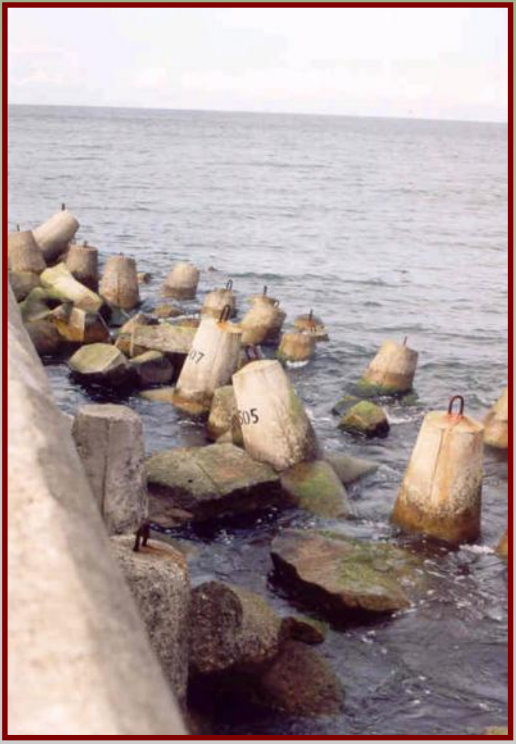
CRS
LUBIATOWO
WLADYSLAWOWO
& HEL PENINSULA
N
BALTIC SEA
ŁEBA
HEL
USTKA
GDYNIA
Gulf of
Gdańsk
GDAŃSK
(IBW PAN)
KOŁOBRZEG
Pomeranian
Bay
ŚWINOUJŚCIE
SZCZECIN

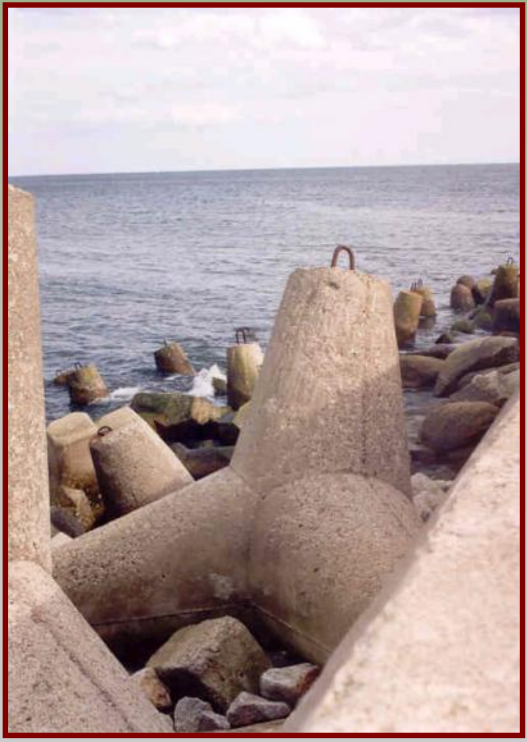
Władysławowo harbour: accretion on west side (bottom), lee side protected by groins and beach fills (top left hand side)



Sand trap at Wladyslawowo harbour

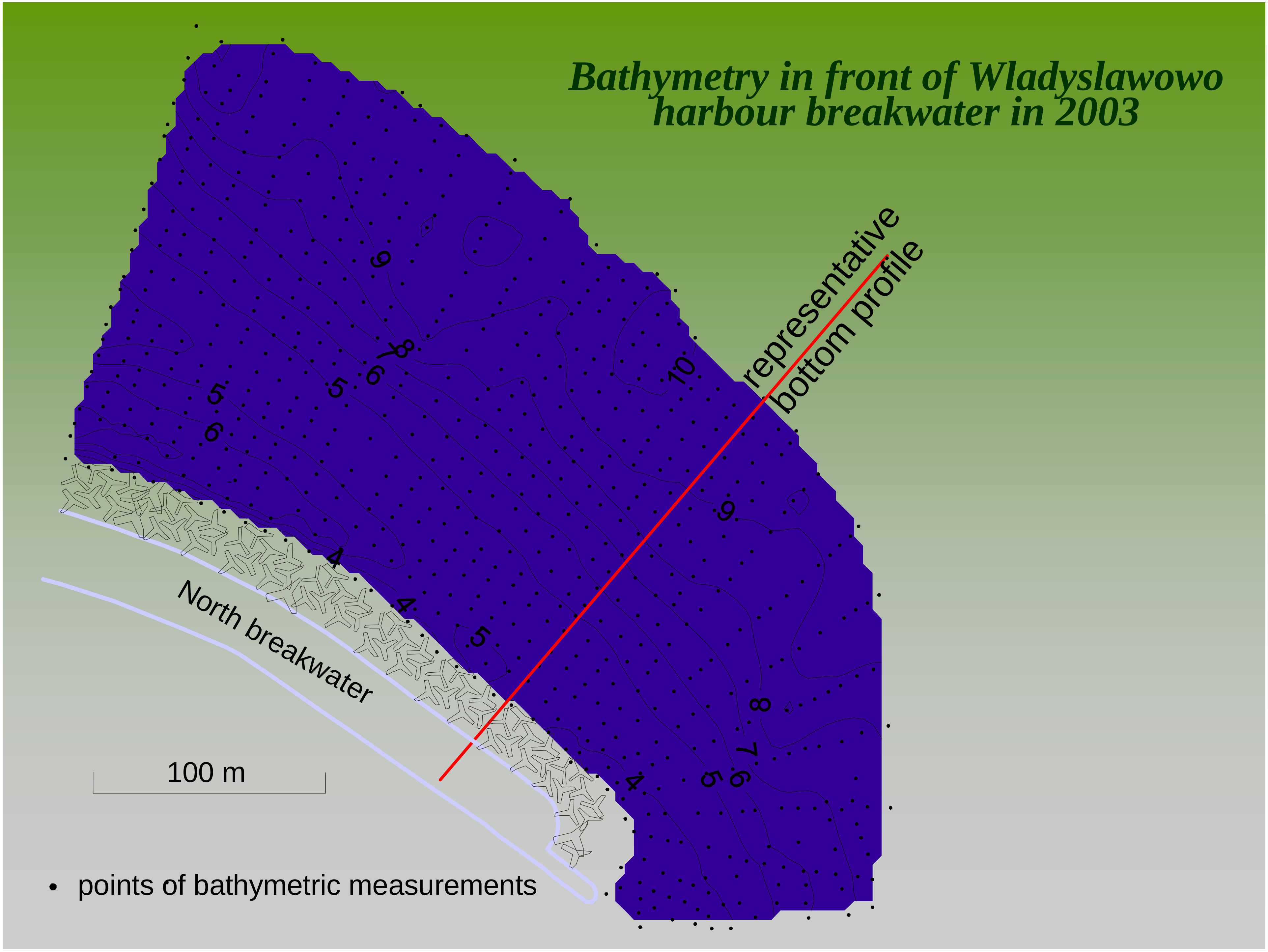
Artificial beach nourishment at Hel Peninsula





Failure of breakwater armour made of tetrapods

Reconstruction of armour at Wladyslawowo harbour breakwater in September 2003



Bathymetry in front of Wladyslawowo
harbour breakwater in 2003
representative
bottom profile
North breakwater
100 m
points of bathymetric measurements
4
5
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8
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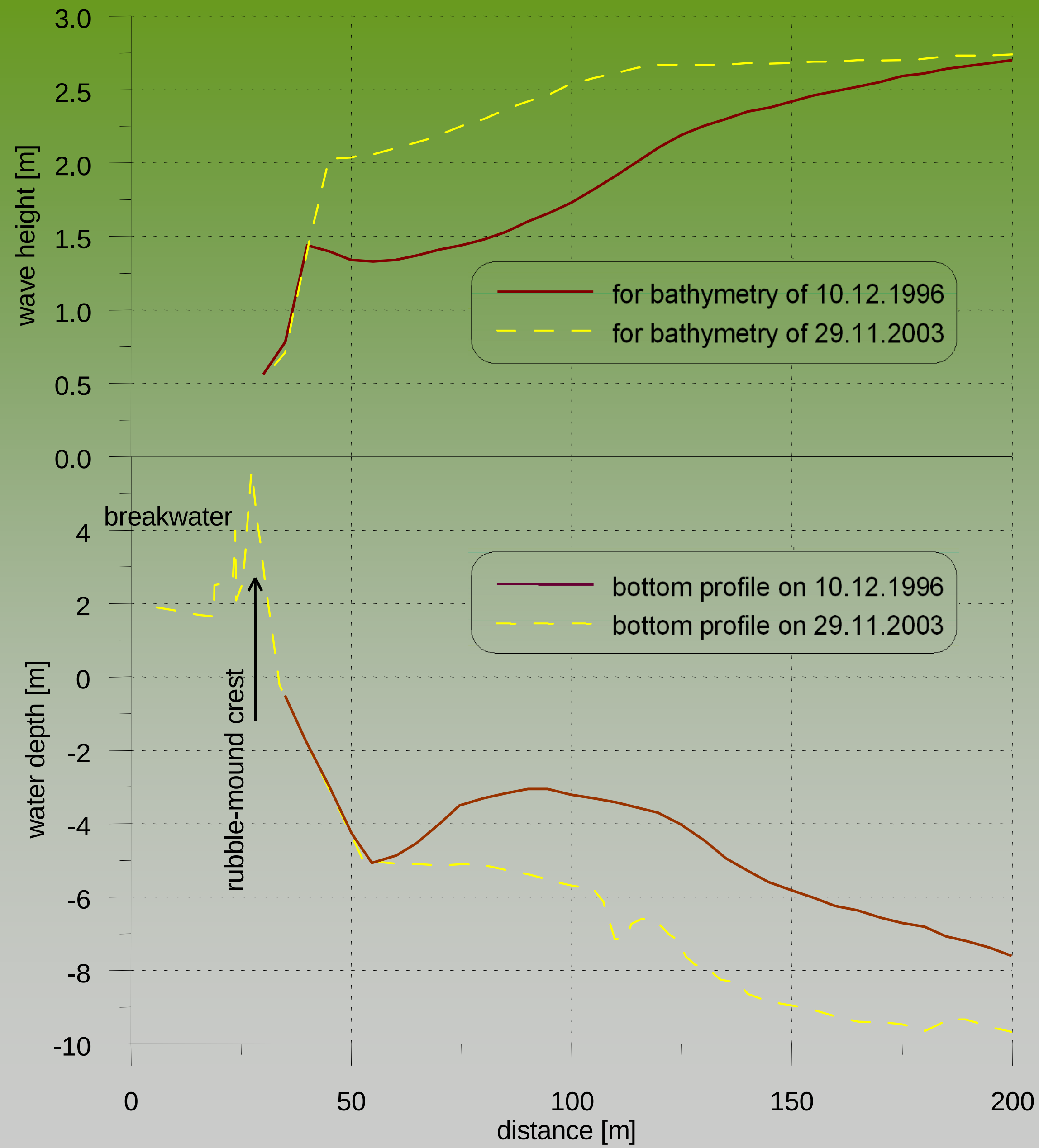
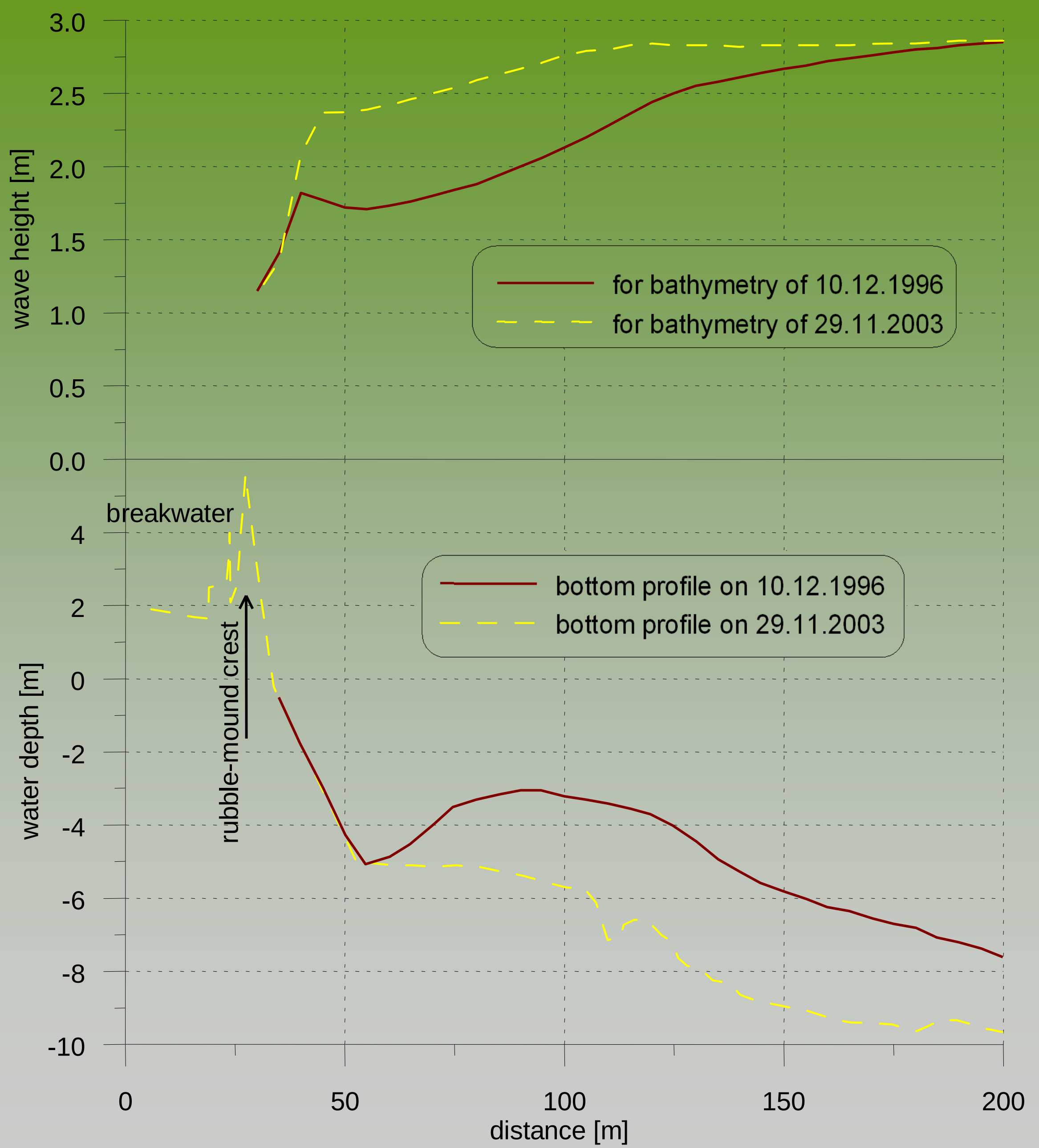
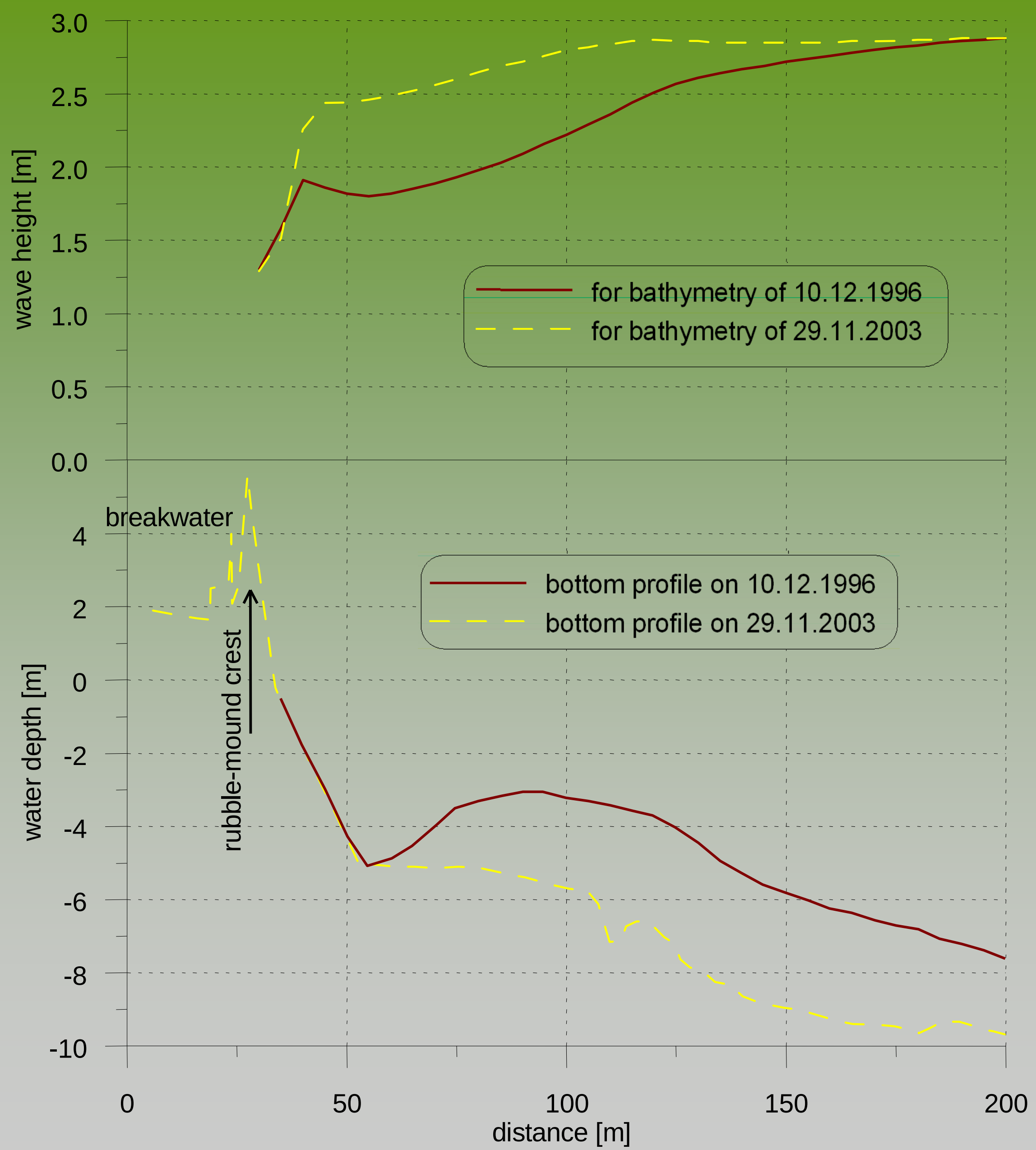


Wave transformation on seabed profiles of December 1996 and November 2003 for normal water level



Wave transformation on seabed profiles of December 1996 and November 2003 for storm surge +1.21 m



Wave transformation on seabed profiles of December 1996 and November 2003 for storm surge +1.51 m

Wave transformation on seabed profile of November 2003 for storm surges +1.21 m, 1.51 m and 1.81 m

$$W = \frac{w_r H^3}{K_D (S_r - 1)^3 \cot \theta}$$

where

W = weight in pounds of an individual armor unit in the primary cover layer. (When the cover layer is two quarry stones in thickness, the stones comprising the primary cover layer can range from about 0.75 W to 1.25 W with about 75 percent of the individual stones weighing more than W. The maximum weight of individual stones depends on the size or shape of the unit. The unit should not be of such a size as to extend an appreciable distance above the average level of the slope.)

w_r = unit weight (saturated surface dry) of armor unit, lbs./ft^3,

H = design wave height at the structure site in feet. (See Section 7.372.),

S_r = specific gravity of armor unit, relative to the water at the structure, ($S_r = w_r/w_w$),

w_w = unit weight of water, fresh water = 62.4 lbs./ft^3, sea water = 64.0 lbs./ft^3,

θ = angle of structure slope measured from horizontal in degrees,

and

K_D = stability coefficient that varies primarily with the shape of the armor units, roughness of the armor unit surface, sharpness of edges and degree of interlocking obtained in placement. (See Table 7-7.)

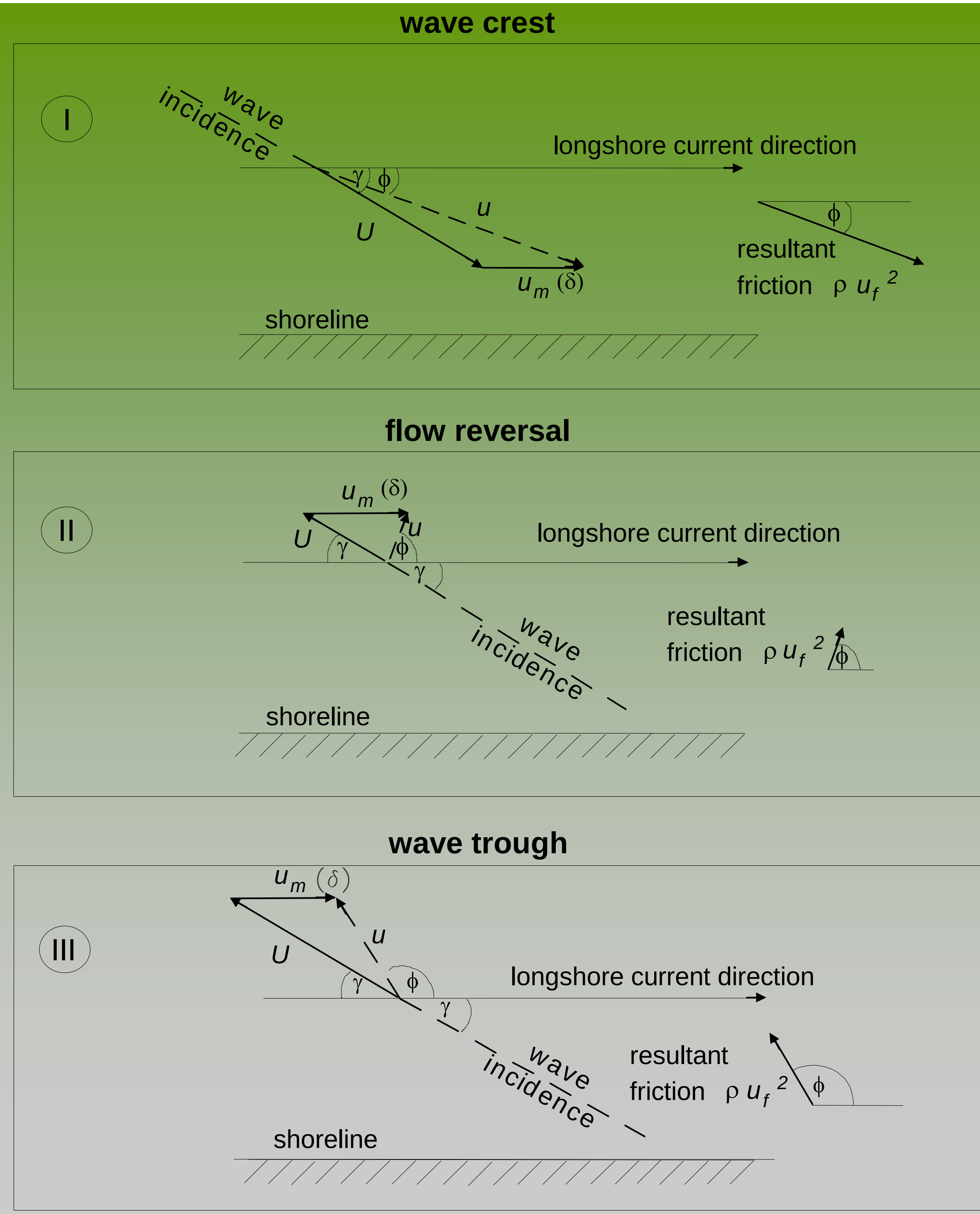
Table 7-7. Suggested K_D Values for Use in Determining Armor Unit Weight

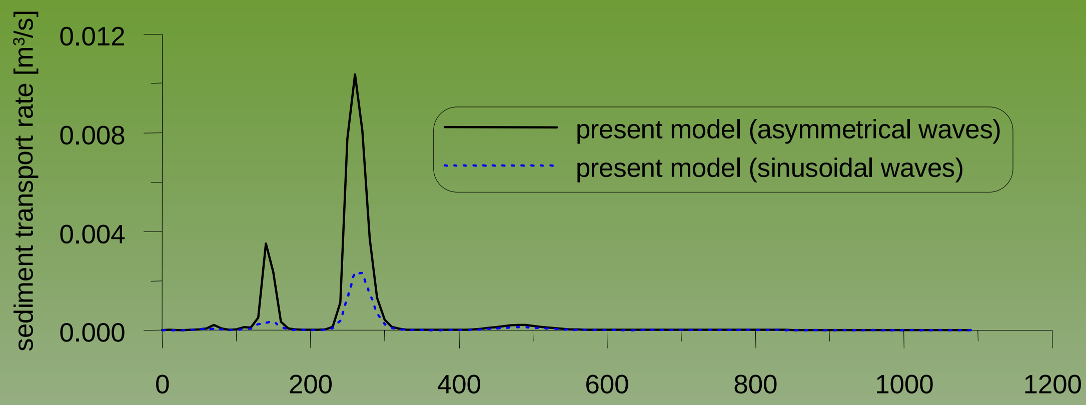
No-Damage Criteria and Minor Overtopping							
Armor Units	n *	Placement	Structure Trunk		Structure Head		
			K_D §		K_D		Slope
			Breaking wave	Nonbreaking wave	Breaking wave	Nonbreaking wave	cot θ
Quarrystone							
Smooth rounded	2	random	2.1	2.4	1.7	1.9	1.5 to 3.0
Smooth rounded	>3	random	2.8	3.2	2.1	2.3	‖
Rough angular	1	random †	†	2.9	†	2.3	‖
Rough angular	2	random	3.5	4.0	2.9 2.5 2.0	3.2 2.8 2.3	1.5 2.0 3.0
Rough angular	>3	random	3.9	4.5	3.7	4.2	‖
Rough angular	2	special ‡	4.8	5.5	3.5	4.5	‖
Tetrapod and Quadripod	2	random	7.2	8.3	5.9 5.5 4.0	6.6 6.1 4.4	1.5 2.0 3.0
Tribar	2	random	9.0	10.4	8.3 7.8 7.0	9.0 8.5 7.7	1.5 2.0 3.0
Dolos	2	random	22.0	25.0	15.0 13.5	16.5 15.0	2.0 ¶ 3.0
Modified Cube	2	random	6.8	7.8	——	5.0	‖
Hexapod	2	random	8.2	9.5	5.0	7.0	‖
Tribar	1	uniform	12.0	15.0	7.5	9.5	‖
Quarrystone (K_{RR}) Graded angular	–	random	2.2	2.5			

Sea level	$H_{design} = H_{10\%}$ [m]	Mass* [kg]
Extreme 20-year storm surge (+1.21 m)	4.65	~8000
Extreme 20-year storm surge + anticipated sea level rise by 0.3 m (+1.51 m)	4.76	~8600
Extreme 20-year storm surge + anticipated sea level rise by 0.6 m (1.81 m)	4.88	~9250

* originally designed as 5000 kg

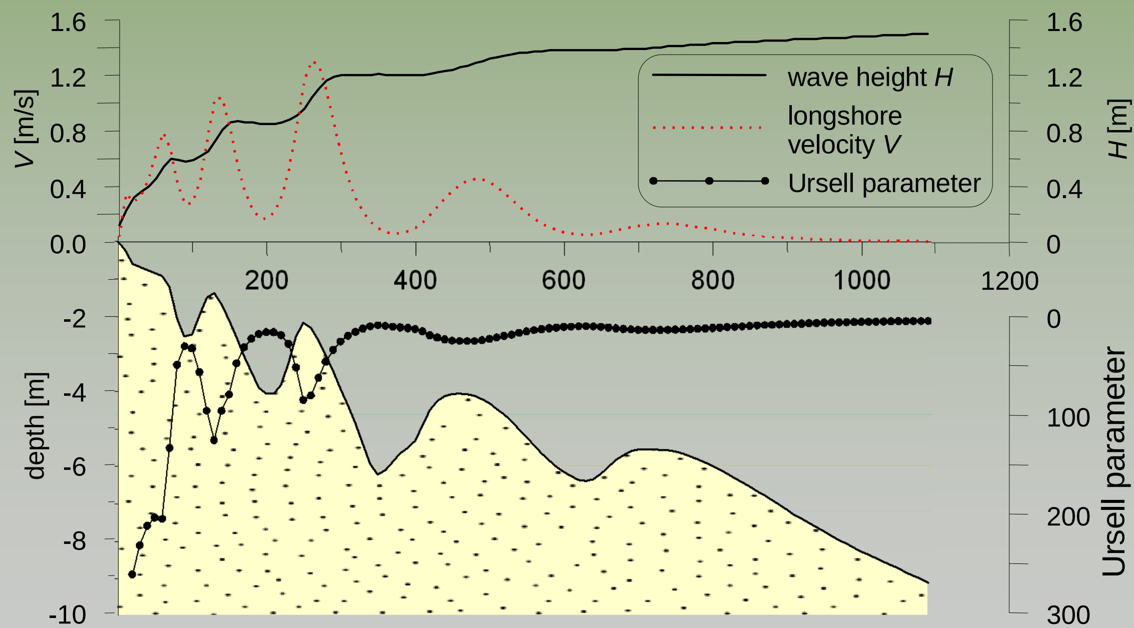
Schemes of interaction of waves and longshore current in the bed boundary layer of the coastal zone





Total longshore sediment transport rate

Approach	IBW PAN model	
	asymmetrical waves	sinusoidal waves
Rate [m^3/s]	0.426	0.114



Thank you for attention